S.E (COMPUTER& INFOR. TECH.) (SEM III) (CBSGS)
Applied Mathematics III

25th November, 2013
3.00 pm to 6.00 pm

shilpa-2nd half-(c)13-33

Con. 7854–13. **GX–12040**

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **three** questions from Q. **2** to Q. **6**.
(3) **Each** question carry **equal** marks.
(4) **Non-programmable** calculator is **allowed**.

1. (a) Find $L^{-1}\left\{\dfrac{e^{4-3s}}{(s+4)^{5/2}}\right\}$ 5

(b) Find the constant a,b,c,d and e If. 5
$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i\,(4x^3y - exy^3 + 4xy)$ is analytic.

(c) Obtain half range Fourier cosine series for $f(x) = \sin x,\ x \in (0, \pi)$. 5

(d) If r and $\bar{r}$ have their usual meaning and a is constant vector, prove that 5

$$\nabla \times \left[\frac{a \times \bar{r}}{r^n}\right] = \frac{(2-n)}{r^n}a + \frac{n\,(a \cdot \bar{r})\,\bar{r}}{r^{n+2}}$$

2. (a) Find the analytic function $f(c) = u + iv$ If $3u + 2v = y^2 - x^2 + 16xy$. 6

(b) Find the z - transform of $\left\{a^{|k|}\right\}$ and hence find the z - transform of $\left\{\left(\frac{1}{2}\right)^{|k|}\right\}$ 6

(c) Obtain Fourier series expansion for $f(x) = \sqrt{1 - \cos x}$, $x \in (0, 2\pi)$ and hence 8

deduce that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$.

3. (a) Find :–

(i) $L^{-1}\left\{\dfrac{s}{(2s+1)^2}\right\}$ 3

(ii) $L^{-1}\left\{\log \dfrac{s^2 + a^2}{\sqrt{s+b}}\right\}$ 3

(b) Find the orthogonal trajectories of the family of curves $e^{-x} \cos y + xy = \propto$ 6
where $\propto$ is the real constant in xy - plane.

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(c) Show that $\vec{F}=\left(ye^{xy}\cos z\right)i+\left(x\,e^{xy}\cos z\right)j-\left(e^{xy}\sin z\right)k$ is irrotational and find the scalar potential for $\vec{F}$ and evaluate $\int_c \overline{F}\cdot dr$ along the curve joining the points (0, 0, 0) and (–1, 2, π). **8**

4. (a) Evaluate by Green's theorem. $\int e^{-x}\sin y\,dx+e^{-x}\cos y\,dy$ where c is the rectangle whose vertices are (0, 0) (π, 0) $(\pi,\frac{\pi}{2})$ and $\left(0,\frac{\pi}{2}\right)$. **6**

(b) Find the half range sine series for the function. **6**

$$f(x)=\frac{2kx}{\ell},\qquad 0\le x\le \ell/2$$
$$=\frac{2k}{\ell}(\ell-x),\quad \frac{\ell}{2}\le x\le \ell$$

(c) Find the inverse z-transform of $\frac{1}{(z-3)(z-2)}$ **8**

(i) $|z|<2$
(ii) $2<|z|<3$
(iii) $|z|>3$.

5. (a) Solve using Laplace transform. $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{-x}$, y (0) = 1, y' (0) = 1. **6**

(b) Express $f(x)=\frac{\pi}{2}e^{-x}\cos x$ for $x>0$ as Fourier sine integral and show that **6**

$$\int_0^\infty \frac{w^3\sin wx}{w^4+4}\,dw=\frac{\pi}{2}e^{-x}\cos x\cdot$$

(c) Evaluate $\iint_s F\cdot n\,ds$, where $\vec{F}=xi-yj+\left(z^2-1\right)k$ and s is the cylinder formed by the surface z = 0, z = 1, $x^2+y^2=4$, using the Gauss - Divergence theorem. **8**

6. (a) Find the inverse Laplace transform by using convolution theorem. 6

$$L^{-1}\left\{\frac{s^2+2s+3}{\left(s^2+2s+5\right)\left(s^2+2s+2\right)}\right\}.$$

(b) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point (1, 1, –1) in the direction towards the point (–3, 5, 6). 6

(c) Find the image of the circle $x^2 + y^2 = 1$, under the transformation $w = \dfrac{5-4z}{4z-2}$. 8

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